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The Onia and the Hybrids of the Exotic Colored Fermions

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Abstract

The spectra and the basic properties of the onia and the hybrids of the exotic colored fermions, which occur in some of the composite models and certain theoretical models, are studied sytematically, based on the QCD and bag model considerations.

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I. Introduction

Leptons and quarks are in color singlets and triplets respectively. However, certain exotic colored fermions which are in different color multiplets are predicted by certain theoretical models. For example, color octet glunios ($\tilde{g}$) by SUSY QCD [1-3], some sextet and/or octet colored fermions by some fermionic composite models [4-7]. Even in the technicolor and extended technicolor model [8-11], exotic colored fermions may occur too, provided that the technicolor force is of that to make technicolor singlet needs odd technifermions, e.g., the technicolor is of $SU(N)$ with odd N.

In fact, let us consider a fermionic model, involving one more nonabelian gauge interaction besides the color among the fundamental fermions, and let the "extra" interaction is stronger than that of the color(being a nonabelian interaction, the "extra" interaction will have asymptotic free and confinement nature), then owing to the confinement, certain kinds of composite particles of the strong gauge interaction, or say bound states , will be formed in the model. They are in singlet of the strong interaction as required by confinement, but may be in various color multiplets even though the fundamental fermions are in color triplets only. Therefore the exotic colored fermions happen to come into a model is a frequent consequence ,especially, for those kinds of models, in which one more strong interaction is involved. Recently, quite a lot of literatures spotlight on the various consequences of the exotic colored fermions [12-14], that in some sense also implies the exotic colored fermion idea is becoming popular.

In composite models, in order to protect the usual composite fermions (leptons and quarks) from obtaining masses in the magnitude order of the strong interaction, as well as, in the technicolor and extended technicolor model, in order to generate some of the wanted goldstones through certain symmetry breaking pattern, most of these models bear some global symmetries, which will not only do something on the usual fermions (quarks and leptons), but also on the exotic colored fermions, e.g., in some composite models, the exotic colored fermions occur in certain solutions of the anomaly match conditions [15-16], hence, if those kinds of solutions will be chosen, the exotic colored fermions may not be very heavy. Comparing them with the composite energy scale, they are considered being massless. On the other hand, there are some arguments [17-19], that if these kinds of models come into reality, i.e., the exotic colored fermions do exist, then based on some consideration about dynamical symmetry breaking, the exotic fermions cannot be as light as quarks and leptons either. They should have masses in the magnitude order of a few hundreds GeV to TeV. This argument is not dependent on specific concerned models very much, but essentially on the color behavior of the exotic fermion themselves (we will explain it briefly in the next section). Having a mass of a few hundreds GeV to TeV, the exotic colored fermions are accessible in near-future experiments.

If the exotic color fermion really exists, to explore the properties and the roles of their bound states would become a very interesting topic.

In this paper, we would focus our discussions on the spectra, the possible decays and some basic properties, for the onia and the hybrids, because they are comparatively easier to deal with and more important. We would rather discuss Dirac exotic fermions than Majorana ones later on, because they appear in many models and there have been some discussions on gluino (Majorana) already.[36] Not losing generality, we will also restrict ourselves to discuss two kinds of the hypothetical exotic fermions S and O in color sextet and octet respectively. [2]

Using the potential framework to analyze the onia and hybrids of heavy quarks is quite successful [20-26]. Thus, for the preliminary discussions, we work in the potential framework, and, based on the perturbative QCD and bag model considerations, generalize the potentials from those for the quarks to those for the exotic colored fermions.

We organize the paper as follow, in the next section, we will briefly review the arguments on the masses of the exotic colored fermions and work out the potentials so as to calculate the spectrum of the onia and the hybrids. In section III, we will discuss their decays and their basic properties. Some conclusions and a summary will be made in the last section on the onia and the hybrids of the exotic fermions.

[2] In fact,there is no stable colored octet and singlet quark, the octet one must be a lepton and the sextet a quark.

II. Spectroscopy

A. The masses of the exotic colored fermions.

In the introduction we pointed out that the masses of the exotic colored fermions will probably be in a magnitude order of a few hundreds GeV to TeV, and it is essentially dependent on the color behavior of the exotic fermions. Now, we briefly retell the key points here, which can be found in refs. [17,18].

Because we are interested only in those fermions which are accessible, in other words they are protected by some symmetry from obtaining a mass in the magnitude order of the confinement energy scale Λ_H of the strong gauge interaction, we will start the story with massless exotic colored fermions. Therefore, let us see how the massless exotic colored fermions acquire masses by certain spontaneous symmetry breaking. It is a consequence of the following two points. One is that below the confinement energy scale Λ_H of a fundamental gauge interaction, the effective interactions among the composite fermions might be described by an effective Lagrangian

$$L_{eff} \sim g^2/\Lambda_H^2 \left(\bar{\psi}\psi\bar{\psi}\psi + \bar{\psi}\Gamma_i\psi\bar{\psi}\Gamma_i\psi\right) , \tag{1}$$

where g is the coupling constant of the fundamental gauge interaction, ψ the composite field operator, and Γ_i certain combinations of the Dirac γ-matrices. The second point is of the nonperturbative effects of QCD, i.e., the composite fermions condense $<\psi\bar{\psi}>\neq 0$. These two points cowork; the first term of eq. (1) will turn out to be the mass term:

$$\begin{aligned} g^2/\Lambda_H^2(\bar{\psi}\psi\bar{\psi}\psi) &\longrightarrow g^2/\Lambda_H^2\bar{\psi} <\psi\bar{\psi}> \psi \sim m\psi\bar{\psi} \\ m &\sim g^2/\Lambda_H^2 <\psi\bar{\psi}> \end{aligned} \tag{2}$$

Marciano [27] first conjectured that to form condensates is dependent on the representation of the color of the fermions, and the conjecture has been proved later by Kogut et al. [28] with the Mont Carlo calculations on the lattice gauge model. They presented the criterion for forming condensate of various fermions due to color interaction :

$$C_2\alpha_s \sim Const.\ (0.5 - 0.6) \ . \tag{3}$$

Here, C_2 is the second order Casimier operator of color, and α_s is the running coupling constant of QCD. The eigenvalue of C_2 is varying from one color multiplet to another, for instance:

$$C_2 = \begin{cases} 0 & singlet \\ 4/3 & triplet \\ 10/3 & sextet \\ 3 & octet \end{cases} \tag{4}$$

and α_s runs with the energy scale changing in logarithm. Therefore, for various multiplets to satisfy the criterion e.g. (3), C_2 changes a little, the α_s too, but the condensate energy scale would change greatly. In general, for condensate we have

$$<\bar{\psi}\psi>\sim \Lambda_c^3 \quad , \tag{5}$$

here Λ_c is the energy scale of the condensate, thus the fermions acquire masses in the order of $g^2\Lambda_c^3/\Lambda_H^2$

$$m \sim g^2/\Lambda_H^2 < \psi\bar{\psi} > \sim g^2\Lambda_c^3/\Lambda_H^2 \ . \tag{6}$$

According to the criterion, eq. (3), it is easy to estimate Λ_c of triplet and sextet and/or octet etc. Because this kind of masses for the usual quarks is about 300 MeV, as a direct inference, for sextet or octet the colored fermion masses should be about a few hundreds GeV to TeV, depending on how many fermions and how heavy their masses, or in other words to say when they begin to play roles in α_s running.[3]

Being general and typical enough, let us consider S and O i.e., the two kinds of the hypothetical exotic colored fermions in a sextet and an octet of color representations and their masses in the region of a few hundreds GeV.

B. The observable states (color singlet) of the exotic colored fermions S and O

Referred to the usual quark cases, we may divide the color singlet bound states involving S and O, into the following categories:

$$\text{Onia} \ : \ S\bar{S}\ , O\bar{O}\ ,$$

[3] It should be noted that here we are concerning the constituent masses in some sense, and ignore the generation mixing problem at all.

$$\begin{aligned} \text{Hybrids} \ &: \ S\bar{S}g\ , Og\ , O\bar{O}g \cdots \\ \text{``Open'' Exotic Hadrons} \ &: \ S\bar{q}\bar{q}\ , Oq\bar{q} \cdots \\ &\quad Sqq\ , Sqq\bar{q}\ , Oqqq\ , O\bar{q}\bar{q}\bar{q} \cdots \\ &\quad SOq\ , S\bar{O}q\ , S\bar{O}\bar{q}\bar{q} \cdots \ . \end{aligned} \tag{7}$$

Of them, the onia and hybrids are comparatively simple for dealing with. We, therefore, highlight on the onia and the hybrids in this moment. Especially the onia are the easiest ones. Based on the knowledge of the quarkonia, it is reasonable to believe the nonrelativistic potential framework is suitable, and as the quarkunium cases, the behavior of the potentials at short distances will be determined by the perturbative QCD precisely. The cases for hybrids are complicated a little, but the Born–Oppenheimer approximation seems to work well owing to the exotic fermions are certainly heavier than that of a real gluon, and the behavior of the exotic fermion interaction at short distances should also be determined by the perturbative QCD.

C. Potential and spectroscopy (Onia)

For quarkumium cases, the potential is obtained based on the following three considerations.

1. At long distances, it grows linearly leading to confinement.
2. At short distances, it approaches what the two-loop perturbative QCD calculation predicts.
3. It turns out the correct spectra of the discovered quarkonia ($c\bar{c}$) and ($b\bar{b}$).

In order to satisfy these three points, there are several versions of the potential (for quarkonia) already[21–26]. In this paper, we would start with one of them given by Igi and Ono[26] as follows:

$$\begin{aligned} V(r) &= V_{AF}(r) + ar \\ V_{AF}(r) &= -\frac{16\pi}{25}\frac{1}{rf(r)}\left(1 + \frac{2\gamma_E + \frac{53}{75}}{f(r)} - \frac{462\ln(f(r))}{625f(r)}\right) \\ f(r) &= \ln\left[1/(\Lambda_{\overline{ms}}r)^2 + b\right] \end{aligned} \tag{8}$$

and the parameters

$$\Lambda_{\overline{ms}} = 300MeV, a = 0.1414GeV^2, b = 19.0,$$

where γ_E is the Euler's constant. The first term of $V(r)$, $V_{AF}(r)$ includes the total short distance behavior, which is determined by QCD two-loop calculation, the second term of $V(r)$, ar presents the long distance behavior of confinement.

For the exotic fermionia we should obviously stand on the same foot as the quarkonia to compute the potential but the fermions are in different color multiplets should be concerned. Therefore, if there is no new dynamics at such short distences which have never been touched by heavy quarkonia, corresponding to eq.(8), the part V_{AF} of the potential should change into

$$\begin{aligned}
V_{AF}(r) &= \frac{\alpha_s(\vec{T}_1 \cdot \vec{T}_2)}{r} , \\
\alpha_s(r) &= \frac{4\pi}{b_0 f(r)}\left(1 + \frac{c}{f(r)} - \frac{b_1 \ln(r)}{b_0^2 f(r)}\right) , \\
b_0 &= 11 - \frac{2}{3}n_f , \\
b_1 &= \frac{34}{3}(C_2(G))^2 - \frac{10}{3}C_2(G)n_f - 2C_2(R)n_f, \\
c &= \frac{1}{b_0}\left(\frac{31}{9}C_2(G) - \frac{10}{9}n_f\right) + 2\gamma_E ,
\end{aligned} \tag{9}$$

where $\vec{T}_1, \vec{T}_2$ are the color operators of the exotic fermion and (anti)fermion respectively. In fact, from $SU(3)$ group theory, we know

$$\vec{T}_1 \cdot \vec{T}_2 = \sum_a T_1^a T_2^a = \frac{(\vec{T}_1 + \vec{T}_2)^2 - \vec{T}_2^2 - \vec{T}_2^2}{2} = \frac{C_2(1,2) - C_2(1) - C_2(2)}{2} \tag{10}$$

the eigenvalue of $(\vec{T}_1 \cdot \vec{T}_2)$ is dependent on the representations of the fermion, (anti)fermion and the system *i.e.* $C_2(1), C_2(2)$ and $C_2(1,2)$. So if the system is in a color singlet and $C_2(1) = C_2(2)$ *i.e.* , the case of onia, then

$$\left(\vec{T}_1 \cdot \vec{T}_2\right) = -C_2(1) = -C_2(2) , \tag{11}$$

that means between the exotic fermion and the antifermion there is a stronger attractive interaction than that between quark and antiquark as $\left(\vec{T}_1 \cdot \vec{T}_2\right) = -\frac{4}{3}$ for qurkonia, $(\vec{T}_1 \cdot \vec{T}_2) = -3$ for $(O\bar{O})_{\underset{\sim}{1}}$ and $(\vec{T}_1 \cdot \vec{T}_2) = -10/3$ for $(S\bar{S})_{\underset{\sim}{1}}$. Later on we will use the foot indices such as $\underset{\sim}{1}$, $\underset{\sim}{8}$ and $\underset{\sim}{10}$ etc. to note the dimension of the color representation.

With respect to ar, the long distant part of V(r), being short of methods to dealing with the nonperturbative effects of QCD, we do not know much about it we consider the following two cases.

(i) color dependent confining potential (CDCP)

$$V(r) = V_{AF}(r) + a\,(\vec{T}_1 \cdot \vec{T}_2)/(-4/3)\ r$$

(ii) universal confining potential (UCP)

$$V(r) = V_{AF}(r) + ar$$

We point out here that because of the exotic fermions being heavy enough the short distance behaviour of the potential becomes crucial namely the low lying energy levels are not dependent on this assumption very much, and for more detail study on this assumption see Ghosh et al[29].

Not knowing the exotic fermion masses exactly and in order to have pictures on the spectra of the onia and the hybrids, we will take some typical values such as 50 GeV, 200 GeV and 1 TeV as examples to discuss in this paper. By using the potentials given above, the energy levels of $O\bar{O}$ and $S\bar{S}$ are computed out (table 1).

The potentials of $(S\bar{S})_{\underset{\sim}{1}}$ and $(O\bar{O})_{\underset{\sim}{1}}$ are compared with that of quarkonium in Fig.1. The formers are much steeper than the latter due to the color factors, so that we have much larger energy level splitting for $(O\bar{O})_{\underset{\sim}{1}}$ and $(S\bar{S})_{\underset{\sim}{1}}$ than those for $(Q\bar{Q})_{\underset{\sim}{1}}$.

TABLE 1

Binding energy of the color singlet fermion-antifermion system $(M(f\bar{f})-m_f-m_{\bar{f}})$ (GeV) computed using two kinds of potentials; CDCP and UCP.

(a) $(O_8\bar{O}_8)_1$

	M_0=50 GeV		200 GeV		1000 GeV	
	CDCP	UCP	CDCP	UCP	CDCP	UCP
1S	-7.303	-7.321	-16.579	-16.585	-49.17	-49.17
1P	-3.782	-3.829	-7.492	-7.507	-19.63	-19.63
1D	-2.662	-2.740	-4.988	-5.016	-11.93	-11.94
2S	-3.559	-3.619	-7.084	-7.104	-18.53	-18.54
3S	-2.349	-2.456	-4.632	-4.671	-11.06	-11.07
4S	-1.631	-1.785	-3.506	-3.567	-7.88	-7.90

(b) $(S_6\bar{S}_6)_1$

	M_S=50 GeV		200 GeV		1000 GeV	
	CDCP	UCP	CDCP	UCP	CDCP	UCP
1S	-8.873	-8.857	-20.229	-20.235	-60.351	-60.352
1P	-4.538	-4.589	-9.066	-9.082	-23.975	-23.978
1D	-3.198	-3.284	-6.000	-6.031	-14.513	-14.521
2S	-4.278	-4.343	-8.570	-8.591	-22.643	-22.648
3S	-2.851	-2.970	-5.574	-5.616	-13.444	-13.455
4S	-2.020	-2.192	-4.218	-4.282	-9.553	-9.570

D. Potential and spectroscopy (hybrid)

Next we consider the states which include one or two constitutent gluons i.e. the hybrids. One might think that the spectra of $((O\bar{O})_8 g)_1$ and $((O\bar{O})_{27} gg)_1$ will not be much different from that of $(O\bar{O})_1$ state since the energy of a gluon is much smaller than that of $(O\bar{O})$ system. However, the behaviour of the potential $V(r)$ for $(O\bar{O})$ or $(S\bar{S})$ system, is proportional to $(\vec{T}_1 \cdot \vec{T}_2)$ and its eigenvalues are different from each other for $(O\bar{O})_1, (S\bar{S})_1, (O\bar{O})_8, (S\bar{S})_8, (O\bar{O})_{10}$, $(O\bar{O})_{27} and (S\bar{S})_{27}$. In addition, for hybrids the interaction between the exotic fermion and antifermion is affected by the consitituent gluon(s).

The effective interaction between the exotic fermion and antifermion S and $\bar{S}$ (or O and $\bar{O}$) can be computed in a specific model. We accept the bag model of Hasenfratz, Horgan, Kuti and Richard (HHKR)[30] and T. D. Lee.[31] as our working framework.

Now we are dealing with the non-Abelian interaction of color, the gluon exchange interaction is always to contribute a factor $g_s^2(\vec{T}_1 \cdot \vec{T}_2)$ instead of an abelian charge for the Abelian case, here g_s is the coupling constant of the color. Being a trick, we may introduce two types of Abelian "charges" I and Y denoted by e and g respectively to deal with the factor $(\vec{T}_1 \cdot \vec{T}_2)$ for the rank 2 non-Abelian algebra of the color. In the $(S\bar{S}g)$ system,the pair of $(S,\bar{S})$ must be in a color octet when the whole system is in a color singlet, then their charge e and g will be assigned in table 2. In table 2 besides $(S\bar{S}g)_1$, the assignments of the charges e and g for $((S\bar{S})_{27}gg)_1, ((O\bar{O})_8 g)_1, ((O\bar{O})_{10}gg)_1$ and $((O\bar{O})_{27}gg)_1$ are also given.

TABLE 2

Abelian charges of type I denoted by e and those of type Y denoted by g for hybrid states

Particle	$S\bar{S}g$: S	$\bar{S}$	g	$(S\bar{S})_{27}gg$: S	$\bar{S}$	g	g
Color	8			27		27	
Charge	6	$\bar{6}$	8	6	6	8	8
e	$\frac{1}{2}\sqrt{\frac{31}{3}}$	$-\frac{1}{2}\sqrt{\frac{31}{3}}$	0	$-\frac{2\sqrt{2}+\sqrt{6}}{3}$	$\frac{2\sqrt{2}-\sqrt{6}}{3}$	$\frac{2\sqrt{6}}{3}$	[illegible]
g	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{2}$	$-\sqrt{3}$	$\frac{-2\sqrt{3}+2}{3}$	$-\frac{2(\sqrt{3}+1)}{3}$	$\frac{\sqrt{3}}{3}$	[illegible]

Particle	$(O\bar{O})_{27}gg$: O	O	g	$\bar{g}$	$(O\bar{O})_{10}gg$: O	O	g	$\bar{g}$
Color	27		27		10		10	
Charge	8	8	8	8	8	8	8	8
e	0	$2\sqrt{\frac{2}{3}}$	$-2\sqrt{\frac{2}{3}}$	0	0	$\sqrt{3}$	$-\sqrt{3}$	0
g	$\sqrt{3}$	$\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{3}}$	$-\sqrt{3}$	$\sqrt{3}$	0	0	$-\sqrt{3}$

Particle	$(OO)_8 g$: O	$\bar{O}$	g
Color / Charge	8	8	8
e	$\frac{3}{2}$	$-\frac{3}{2}$	0
g	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{2}$	$-\sqrt{3}$

The Born-Oppenheimer approximation is used to obtain the effective potential between the exotic fermion and antifermion. Now we take S and $\bar{S}$ as example to consider the $(S\bar{S}g)_{\underline{1}}$ system.

We put $S, \bar{S}$ and one of transverse (constituent) gluon into a spherical bag with radius R. In fact, the spherical bag approximation makes the slope of the potential higher than that of the actual potential, thus, it would be desirable to use a deformed bag. However the calculation with a deformed bag is very complicated. Here for the low lying states, which is deformed from a spherical bag a little, we think that to use the spherical bag approximation is good enough. The constituent gluon occupies a definite quantum number in the spherical bag i.e. stationary TE or TM mode. A straightforward calculation the potential between S and $\bar{S}$ in the $(S\bar{S}g)_{\underline{1}}$ system is obtained (it is defferent from that computed out in ref.[3] due to the various eigenvalue of $(\vec{T}_1 \cdot \vec{T}_2)$.

$$\begin{aligned} V^{8}_{(S\bar{S})}(r) &= -\frac{11}{6}\frac{\alpha_s}{r} + \frac{4}{3}\pi R^3 \Lambda_B^4 + \frac{\alpha^{\lambda}_{nj}}{R} \\ &+ \frac{\alpha_s}{R}\left[\frac{10}{3}\left\{\frac{r^2}{4R^2 - r^2} - \ln\left(1 - \left(\frac{r}{2R}\right)^2\right)\right\}\right. \\ &+ \left.\frac{11}{6}\frac{r^2}{4R^2 + r^2} + \frac{11}{6}\ln\left(1 + \left(\frac{r}{2R}\right)^2\right) - \frac{27}{10} + \frac{3}{8}\frac{r^2}{R^2}\right] \end{aligned} \tag{12}$$

for $((S\bar{S})_{\underline{8}}, g)_{\underline{1}}$. Here the r is the distance between S and $\bar{S}$, the first term comes from the one gluon exchange, the Coulomb - like interaction. The second term is the volume energy and the third term is the energy of the gluon without considering the interaction. We compute only the lowest gluon mode (TE), which corresponds to $\alpha^{TE} = 2.744$.

In the same way we find the other cases of the potentials:

$$\begin{aligned} V^{27}_{(S\bar{S})}(r) &= \frac{2}{3}\frac{\alpha_s}{r} + \frac{4}{3}\pi R^3 \Lambda_B^4 + \frac{\alpha^{\lambda}_{nj}}{R} + \frac{\alpha^{\lambda'}_{n'j'}}{R} \\ &+ \frac{\alpha_s}{R}\left[\frac{10}{3}\left\{\frac{r^2}{4R^2 - r^2} - \ln\left(1 - \left(\frac{r}{2R}\right)^2\right)\right\}\right. \\ &+ \frac{2}{3}\left\{-\frac{r^2}{4R^2 + r^2} - \ln\left(1 + \left(\frac{r}{2R}\right)^2\right)\right\} \\ &- \left.\frac{36}{5} + \left(\frac{r}{R}\right)^2\right] \end{aligned} \tag{13}$$

for $((S\bar{S})_{\underline{27}}, gg)_{\underline{1}}$,

$$\begin{aligned} V^{8}_{(O\bar{O})}(r) &= -\frac{3}{2}\frac{\alpha_s}{r} + \frac{4}{3}\pi R^3 \Lambda_B^4 + \frac{\alpha^{\lambda}_{nj}}{R} \\ &+ \frac{3\alpha_s}{2R}\left\{\frac{2r^2}{4R^2 - r^2} - 2\ln\left(1 - \frac{r^2}{4R^2}\right) + \frac{r^2}{4R^2 + r^2}\right. \\ &+ \ln\left(1 + \left(\frac{r}{2R}\right)^2\right) - 3 \\ &+ \left.\frac{r^2}{4R^2} + \frac{6}{5}\right\} \end{aligned} \tag{14}$$

for $((O\bar{O})_{\underline{8}}, g)_{\underline{1}}$,

$$\begin{aligned} V^{10}_{(O\bar{O})}(r) &= \frac{4}{3}\pi R^3 \Lambda_B^4 + \frac{\alpha^{\lambda}_{nj}}{R} + \frac{\alpha^{\lambda'}_{n'j'}}{R} \\ &+ \frac{3\alpha_s}{R}\left\{\frac{r^2}{4R^2 - r^2} - \ln\left(1 - \left(\frac{r}{2R}\right)^2\right)\right. \\ &+ \left.\left(\frac{r}{2R}\right)^2 - \frac{9}{5}\right\} \end{aligned} \tag{15}$$

for $((O\bar{O})_{\underline{10}}, gg)_{\underline{1}}$ and

$$\begin{aligned} V^{27}_{(O\bar{O})}(r) &= \frac{\alpha_s}{r} + \frac{4}{3}\pi R^3 \Lambda_B^4 + \frac{\alpha^{\lambda}_{nj}}{R} + \frac{\alpha^{\lambda'}_{n'j'}}{R} \\ &+ \frac{\alpha_s}{R}\left[\frac{3r^2}{4R^2 - r^2} - \frac{r^2}{4R^2 + r^2} - 3\ln\left(1 - \left(\frac{r}{2R}\right)^2\right)\right. \\ &- \left.\ln\left(1 + \left(\frac{r}{2R}\right)^2\right) + 4\left(\frac{r}{2R}\right)^2 - \frac{36}{5}\right] \end{aligned} \tag{16}$$

for $((O\bar{O})_{\underline{27}}, gg)_{\underline{1}}$.

The bag radius R is determined by the minimization condition $\delta V/\delta R = 0$ for given r. The effective potentials are determined by min $V(R.r)$, which is called $V(r)$ again. The effective potentials found in this way for various color combinations are compared with those for $(S\bar{S})_{\underset{\sim}{1}}$ and $(O\bar{O})_{\underset{\sim}{1}}$. In order to obtain the spectra, let us put the effective potential into the Schrödinger equation for each case, then we have:

$$
\begin{aligned}
-\frac{1}{M}\frac{d^2}{dr^2}\psi(r) \quad &+ \quad \frac{1}{M}\frac{<(\vec{L}-\vec{j})^2)}{r^2}\psi(r) + V(r)\psi = E\psi(r) \\
<(\vec{L}-\vec{j})^2> \quad &= \quad L(L+1) + j(j+1) - 2\Lambda^2 \qquad (17)
\end{aligned}
$$

where $\vec{L}$ is the total orbital angular momentum and $\vec{j}$ is the angular momentum of the gluon ($j = 1$ for the lowest TE mode). Λ is the eigenvalue for the projection of the gluon's angular momentum along the $(S\bar{S})_{\underset{\sim}{8}}$ or $(O\bar{O})_{\underset{\sim}{8}}$ axis.

In fact,there are two models to classify these hybrid states. In the first model we use the similarity for the states to those for diatomic molecules as what did in ref.[32]. We neglect the spin-dependent forces, thus the energy levels can be specified by the value of $<(\vec{L}-\vec{j})^2>$ and the number of radial nodes of the wave function. The quantum numbers of $((S\bar{S})_{\underset{\sim}{8}}g)_{\underset{\sim}{1}}$ states in each level in this model are exactly the same as those for hybrid meson $((Q\bar{Q})_{\underset{\sim}{8}}g)_{\underset{\sim}{1}}$, which are already described in ref.[32] and we will not repeat here. But we need to point out here that for $((O\bar{O})_{\underset{\sim}{8}}g)_{\underset{\sim}{1}}$, there is one very important difference [17] namely when reducing two of coupled octets then two different octets will occur in the decomposite series

$$
\underset{\sim}{8} \times \underset{\sim}{8} = \underset{\sim}{1} + \underset{\sim}{8a} + \underset{\sim}{8s} + \underset{\sim}{10} + \underset{\sim}{10^*} + \underset{\sim}{27} \qquad (18)
$$

the symmetric one $\underset{\sim}{8s}$ and the antisymmetric one $\underset{\sim}{8a}$. Therefore, we have $(O\bar{O})_{\underset{\sim}{8a}}$ and $(O\bar{O})_{\underset{\sim}{8s}}$, they under the first order approximation degenerate, but have opposite charge "parity". In the next section we shall see that $(O\bar{O})_{\underset{\sim}{8a}}$ and $(O\bar{O})_{\underset{\sim}{8s}}$ have very different decay modes and in the last section we will discuss the problem on degenerate removing. The rest quantum numbers of $(O\bar{O})_{\underset{\sim}{8a}}$ and $(O\bar{O})_{\underset{\sim}{8s}}$ are the same as those of $(S\bar{S})_{\underset{\sim}{8}}$.

Following ref.[32] we name the energy levels in the following way according to the

expectation values of $<(\vec{L}-\vec{j})^2>$

$$
<(\vec{L}-\vec{j})^2> = \begin{cases} 2, & 1A, \quad 2A, \quad 3A, \cdots \\ 4, & 1B, \quad 2B, \quad 3B, \cdots \\ 6, & 1C, \quad 2C, \quad 3C, \cdots \end{cases} \qquad (19)
$$

We do not know for sure if the above model is appropriate for $(S\bar{S}g)_{\underset{\sim}{1}}$ or $(O\bar{O}g)_{\underset{\sim}{1}}$ states and if these hybrid states are not so similar to the diatomic molecules. In the case of the light diatomic molecule, the motion of the electrons are very much restricted to the axis of the two nuclei, so that it is better to introduce a coordinate system which is moving together with the two nuclei. In contrary to this aspect, for hybrid states the bag radia are much larger than the distances between S and $\bar{S}$ (or O and $\bar{O}$), especially for $((S\bar{S})_{\underset{\sim}{8}}g)_{\underset{\sim}{1}}$ (or $((O\bar{O})_{\underset{\sim}{8}}g)_{\underset{\sim}{1}}$) case, the interaction of the two constituents is attractive at short distances (Fig. 1). This means $(S\bar{S})_{\underset{\sim}{8}}$ (or $(O\bar{O})_{\underset{\sim}{8}}$) is well concentrated near the center and the wave function of the gluon is not much disturbed by the finite size of $(S\bar{S})_{\underset{\sim}{8}}$ (or $(O\bar{O})_{\underset{\sim}{8}}$) wave function. Therefore, the $(S\bar{S})_{\underset{\sim}{8}}$ (or $(O\bar{O})_{\underset{\sim}{8}}$) axis does not play an important role for the gluon wave function. If the picture is true not only due to the attractive force at short distances but also due to heavy masses of the constituents, then $((S\bar{S})gg)_{\underset{\sim}{1}}$ (or $((O\bar{O})gg)_{\underset{\sim}{1}}$) states might be the case too.

In the second model for the hybrid state classification it isreasonable to assume that the gluon inside the bag is free.

The total wave function is simply the product of the $(S\bar{S})_{\underset{\sim}{8}}$ (or $(O\bar{O})_{\underset{\sim}{8}}$) wave function (with $\vec{L}^2$ instead of $<(\vec{L}-\vec{j})^2>$ in eq.(17) and shifting zero point of energy) and the free gluon wave function. In this model the lowest state for $(S\bar{S})$ or $(O\bar{O})$ is simply $1S$ state and next one is the $1P$ state and so on. Owing to $<(\vec{L}-\vec{j})^2> \geq 2$ for the TE model in the first model, the $(S\bar{S})_{\underset{\sim}{8}}$ or $(O\bar{O})_{\underset{\sim}{8}}$ cannot be in an S state in the sense. Energy levels in two models are compared in the following

$<(\vec{L}-\vec{j})^2>$ or $\vec{L}^2$	Model 1	Model 2
0	forbidden	allowed (1S)
2	allowed (1A)	allowed (1P)
4	allowed (1B)	forbidden
6	allowed (1C)	allowed (1D)

Note that the model 1 is constructed for hybrid systems $(F\bar{F}g)_{\underline{1}}$, one gluon hybrids, and can not be applied to the states with two gluons such as $((S\bar{S})gg)_{\underline{1}}$ and $((O\bar{O})gg)_{\underline{1}}$ directly. The diatomic molecule-like model for two gluon hybrid states has not been studied yet. However we think that we do not need such a model so far because probably the hybrids for their low lying states are more likely suitable to the second model.

Each effective potential includes a term $(\vec{F}_1 \cdot \vec{F}_2)\alpha_s/r$ (the first term of eqs. (9), (12) – (16)) and this term correspond to one gluon exchange diagram. As discussed for onia the one gluon exchange potential should not be as singular as $1/r$ but should be softened to $1/(r \ln r)$ due to high order corrections. Thus we replace one gluon exchange term by $V_{AF}(r)$ defined in Eq. (5) after a suitable change of the color factor due to $(\vec{T}_1 \cdot \vec{T}_2)$ for different multiplets.

In table 3 we show spectra of the various hybrid states calculated in this way. Mostly we use the second model but only for $(S\bar{S}g)_{\underline{1}}$ and $(O\bar{O}g)_{\underline{1}}$ states we add states $1A, 1B, 1C$ following the first model classification. As expected the level splitings of the hybrid states are much less than those of onia $(S\bar{S})_{\underline{1}}$ and $(O\bar{O})_{\underline{1}}$ states but still quite big for the low lying states of $((O\bar{O})g)_{\underline{1}}$ and $((S\bar{S})g)_{\underline{1}}$.

TABLE 3

Binding energies of hybrid states $\{M(hybrid) - m_f - m_{\bar{f}}\}$ (GeV)

(a) $(S_6 \bar{S}_6)_8\, g$

STATE	$\ell(\ell+1)$	M_S=50 GeV	200 GeV	1000 GeV
1 S	0	− 2.692	− 6.795	−20.930
1 P=1 A	2	− 1.090	− 2.781	− 8.143
1 B	4	− 0.735	− 2.028	− 5.856
1 D=1 C	6	− 0.534	− 1.656	− 4.755
2 S	0	− 0.965	− 2.591	− 7.640
3 S	0	− 0.336	− 1.479	− 4.350
4 S	0	- 0.053	−0.936	− 2.945

(b) $(O_8 \bar{O}_8)_8\, g$

STATE	$\ell(\ell+1)$	M_O=50 GeV	200 GeV	1000 GeV
1 S	0	− 1.697	− 4.542	− 14.290
1 P=1 A	2	− 0.559	− 1.761	− 5.476
1 B	4	− 0.293	− 1.235	− 3.895
1 D=1 C	6	− 0.139	−0.971	− 3.132
2 S	0	− 0.455	− 1.626	− 5.123
3 S	0	0.030	−0.836	− 2.845
4 S	0	0.338	−0.433	− 1.871

(c) $(O_8 \bar{O}_8)_{10}\, gg$

STATE	$\ell(\ell+1)$	M_S= 50 GeV	200 GeV	1000 GeV
1 S	0	1.552	1.517	1.496
1 P	2	1.601	1.541	1.507
1 D	4	1.647	1.566	1.518
2 S	0	1.646	1.564	1.518
3 S	0	1.738	1.611	1.540
4 S	0	1.829	1.658	1.560

(d) $(O_8 \bar{O}_8)_{27}\, gg$

STATE	$\ell(\ell+1)$	M_S= 50 GeV	200 GeV	1000 GeV
1 S	0	1.826	1.807	1.7966
1 P	2	1.833	1.809	1.7970
1 D	4	1.847	1.812	1.7976
2 S	0	1.903	1.845	1.8137
3 S	0	1.979	1.883	1.8307
4 S	0	2.056	1.922	1.8477

(e) $(S_6 \bar{S}_6)_{27}\, gg$

STATE	$\ell(\ell+1)$	M_S= 50 GeV	200 GeV	1000 GeV
1 S	0	1.722	1.701	1.6902
1 P	2	1.732	1.704	1.6907
1 D	4	1.750	1.709	1.6917
2 S	0	1.801	1.741	1.7081
3 S	0	1.881	1.781	1.7258
4 S	0	1.961	1.821	1.7436

III. The decay properties of the onia and the hybrids

In this section we are to discuss the decay properties of the onia and the hybrids. If the lifetime of the constituent fermions is very short then the constituent fermion decays are dominant the decay aspects of the onia and the hybrids. However, although the lifetime of the exotic colored fermions is model dependent, generally speaking, for any "exotics" there always exits at least one quantum number which is conserved under "strong" interactions, so that the lightest of the exotics is stable under the "strong" interactions and its decay will be only due to "weak" interaction(s) which breaks the quantum number, namely the lightest one among the exotic colored fermions with nonzero value of the quantum number is relatively stable. Therefore in this section we will not discuss the decays due to the constituent directly, that means the resultant aspect obtained in the paper is suitable for those onia and hybrids of the lightest exotic colored fermions.

Although the onia and hybrids always have zero value of the exotic quantum number, due to the nonzero exotic quantum number of the constituents, the onia and the hybrids decay into the usual gluon(s) and/or quark(s) may occur only through annihilation (hidden exotic). Thus the annihilation channels and the transitions among the low lying states of the onia and the hybrids are substantial to determine the onium and the hybrid lifetime. We should note here for most processes the color interaction can be considered relatively weak dut to the coupling running. Namely, for most of the onia and the hybrids, the decay rates into gluon(s) and/or usual quarks may be calculated based on the pertubative QCD framework and we list the important decays and one gluon emitting transitions as well as the related formulas in the following.

A. The two gluon decay channel

The onia and the hybrids decay into two gluons is described by the Feymann diagram (Fig. 2) and the corresponding decay rates are given by the following formulas:

$$\Gamma\left(\left[{}^1S_0(S\bar{S})_{\underline{1}}\right] \to 2g\right) = \frac{25}{3}\frac{\alpha_s^2}{m_S^2}\,|R(0)|^2\ , \tag{20}$$

$$\Gamma\left(\left[{}^1S_0(O\bar{O})_{\underline{1}}\right] \to 2g\right) = 9\frac{\alpha_s^2}{m_O^2}\,|R(0)|^2\ , \tag{21}$$

$$\Gamma\left(\left[{}^3P_2(S\bar S)_{\underline{1}}\right] \to 2g\right) = \frac{640}{27}\frac{\alpha_s^2}{m_S^4}\,|R'(0)|^2 \ , \qquad (22)$$

$$\Gamma\left(\left[{}^3P_2(O\bar O)_{\underline{1}}\right] \to 2g\right) = \frac{128}{5}\frac{\alpha_s^2}{m_O^4}\,|R'(0)|^2 \ , \qquad (23)$$

$$\Gamma\left(\left[{}^1S_0,(S\bar S)_{\underline{8}}\right] \to 2g\right) = \frac{49}{12}\frac{\alpha_s^2}{m_S^2}\,|R(0)|^2 \ , \qquad (24)$$

$$\Gamma\left(\left[{}^1S_0,(O\bar O)_{\underline{8}_s}\right] \to 2g\right) = \frac{9}{2}\frac{\alpha_s^2}{m_O^2}\,|R(0)|^2 \ , \qquad (25)$$

$$\Gamma\left(\left[{}^1S_0,(O\bar O)_{\underline{8}_a}\right] \to 2g\right) = 0 \ , \qquad (26)$$

$$\Gamma\left(\left[{}^3P_2,(S\bar S)_{\underline{8}}\right] \to 2g\right) = \frac{784}{135}\frac{\alpha_s^2}{m_S^4}\,|R'(0)|^2 \ , \qquad (27)$$

$$\Gamma\left(\left[{}^3P_2,(O\bar O)_{\underline{8}_s}\right] \to 2g\right) = \frac{32}{5}\frac{\alpha_s^2}{m_O^4}\,|R'(0)|^2 \ , \qquad (28)$$

$$\Gamma\left(\left[{}^3P_2,(O\bar O)_{\underline{8}_a}\right] \to 2g\right) = 0 \ , \qquad (29)$$

here m_S, m_O are the masses of S and O respectively, $R(0)$ is the radial wave function at origin and $R'(0)$ is the corresponding derivative of the wave function.

B. The quark pair decay channel

The onia and the hybrids decay into quark pair is described by the Feymann diagram (fig. 3) and the corresponding decay rates are given by the following formulas:

$$\Gamma\left(\left[{}^3S_1,(S\bar S)_8\right] \to Q\bar Q\right) = \frac{5}{24}\alpha_s\beta_Q\frac{m_Q^2+2m_S^2}{m_S^4}\,|R(0)|^2 \ , \qquad (30)$$

$$\Gamma\left(\left[{}^3S_1,(O\bar O)_{\underline{8}_a}\right] \to Q\bar Q\right) = \frac{1}{4}\alpha_s\beta_Q\frac{m_Q^2+2m_O^2}{m_O^4}\,|R(0)|^2 \ , \qquad (31)$$

here m_Q is the mass of the quark Q and antiquark $\bar Q, \beta_Q$ is the velocity of the quarks in C.M.S. $\left(\beta_Q = \frac{2k_Q}{\sqrt{s}}\right)$.

We should note here that due to the same mechanism as that for suppressing the neutral flavor change current, the derict contact effective interaction such as described by eq.(1) is very weak, so that in the above eqs. (30) and (31) we ignored its contributions at all.

C. The three gluon decay channel

The onia and the hybrids decay into three gluons is described by the Feymann diagram (Fig. 4) and the corresponding decay rates are given by the following formulas:

$$\Gamma\left(\left[{}^3S_1,(S\bar S)_{\underline{1}}\right] \to 3g\right) = \frac{245}{81\pi}(\pi^2-9)\frac{\alpha_s^3}{m_S^2}\,|R(0)|^2 \ , \qquad (32)$$

$$\Gamma\left(\left[{}^3S_1,(O\bar O)_{\underline{1}}\right] \to 3g\right) = 0 \ . \qquad (33)$$

D. The one gluon emittion channel

The one gluon emittion of hybrids is described by the Feymann diagram (Fig. 5 and the corresponding decay rates are given by the following formulas:

$$\Gamma\left(\left[{}^3S_1,(O\bar O)_{\underline{8}_a}\right] \to \left[{}^1S_0,(O\bar O)_{\underline{1}}\right]+g\right) = \frac{4}{3}C_1\frac{\alpha_s k^3}{m_O^2}|I_0(k)|^2 \ , \qquad (34)$$

$$\Gamma\left(\left[{}^1S_0,(O\bar O)_{\underline{8}_a}\right] \to \left[{}^3S_1,(O\bar O)_{\underline{1}}\right]+g\right) = 4C_1\frac{\alpha_s k^3}{m_O^2}|I_0(k)|^2 \ , \qquad (35)$$

$$\Gamma\left(\left[{}^1S_0,(O\bar O)_{\underline{8}_a}\right] \to \left[{}^1P_1,(O\bar O)_{\underline{1}}\right]+g\right) = \frac{4}{3}C_1\alpha_s k^3|I_1(k)|^2 \ , \qquad (36)$$

$$\Gamma\left(\left[{}^1P_1,(O\bar O)_{\underline{8}_a}\right] \to \left[{}^1S_0,(O\bar O)_{\underline{1}}\right]+g\right) = \frac{4}{9}C_1\alpha_s k^3|I_1(k)|^2 \ , \qquad (37)$$

$$\Gamma\left(\left[{}^3S_1,(O\bar O)_{\underline{8}_a}\right] \to \left[{}^3P_1,(O\bar O)_{\underline{1}}\right]+g\right) = \frac{2J+1}{9}\cdot\frac{4}{3}C_1\alpha_s k^3|I_1(k)|^2 \ , \qquad (38)$$

$$\Gamma\left(\left[{}^3P_J,(O\bar O)_{\underline{8}_a}\right] \to \left[{}^3S_1,(O\bar O)_{\underline{1}}\right]+g\right) = \frac{4}{9}C_1\alpha_s k^3|I_1(k)|^2 \ , \qquad (39)$$

here k is the momentum of the gluon, $I_0(k)$ and $I_1(k)$ are the integrations of the wave functions' overlapping:

$$I_0(k) = \int dr\, r^2\left(R_f(r)j_0\left(\frac{kr}{2}\right)R_i(r)\right) \ , \qquad (40)$$

and

$$I_1(k) = \int dr\, r^2\left(R_f(r)r\, R_i(r)\right) \ . \qquad (41)$$

Due to O in the adjoint representaion of color, it is easy to check that the following processes are forbidden,

$$\Gamma\left((O\bar{O})_{\underset{\sim}{8_s}} \to (O\bar{O})_{\underset{\sim}{1}} + g\right) = 0 \ , \qquad (42)$$

$$\Gamma\left((O\bar{O})_{\underset{\sim}{8_s}} \to (O\bar{O})_{\underset{\sim}{8_s}} + g\right) = 0 \ , \qquad (43)$$

and

$$\Gamma\left((O\bar{O})_{\underset{\sim}{8_a}} \to (O\bar{O})_{\underset{\sim}{8_a}} + g\right) = 0 \ . \qquad (44)$$

For the case of $(S\bar{S})$ system there is no such different states as $(O\bar{O})$ system the symmetric one $\underset{\sim}{8_s}$ and the antisymmetric one $\underset{\sim}{8_a}$, so there is no such selection rules, they may have the processes (34)-(39) and the formulas are the same but the color factor C_1 should change into C_2 and here we have $C_1 = 3/8$ and $C_2 = 5/12$.

If we put the values of the wave functions obtained in Sec.2 by solving the Schrödinger equation with corresponding effective potentials for the onia and the hybrids into the above formulas, the partial width may be calculated immediately. We put the numberical results into the table 4.

TABLE 4 Typical theoretical values of decay rates in (MeV) computed by using the UCP. The results obtained by using the CDCP are very similar to these. Other decay rates can be computed simply by replacing the color factor.

4a) The $(O\bar{O})$ system

M_O (The mass of the fermion)		50 GeV	200 GeV	1000 GeV
α_s		0.1057	0.0885	0.0745
$(1^1S_0,(O\bar{O})_1) \to 2g$		365	570	1110
$(1^3S_1,(O\bar{O})_1) \to 3g$		0	0	0
$(1^3P_2,(O\bar{O})_1) \to 2g$		2.35	1.97	2.34
$(2^1S_0,(O\bar{O})_1) \to 2g$		74.0	98.4	182
$(2^3S_1,(O\bar{O})_1) \to 3g$		0	0	0
$(1^1S_0,(O\bar{O})_{8S}) \to 2g$		182	285	555
$(1^1S_0,(O\bar{O})_{8a})$	$\to (1^3S_1,(O\bar{O})_1)+g$	7.47	4.07	3.41
	$\to (1^1P_1,(O\bar{O})_1)+g$	21.3	4.93	1.53
	$\to (2^3S_1,(O\bar{O})_1)+g$	0.0884	0.0117	0.00186
$(1^3S_1,(O\bar{O})_{8a})$	$\to$ "g" $\to Q\bar{Q}$	3.52	5.27	9.90
	$\to (1^1S_0,(O\bar{O})_1)+g$	2.49	1.36	1.14
	$\to (1^3P_2,(O\bar{O})_1)+g$	11.8	2.74	0.852
	$\to (2^1S_0,(O\bar{O})_1)+g$	0.0295	0.0039	6.19×10^{-4}
$(1^1P_1,(O\bar{O})_{8a})$	$\to (1^1S_0,(O\bar{O})_1)+g$	16.7	12.2	5.14
	$\to (2^1S_0,(O\bar{O})_1)+g$	53.7	27.5	0.0607
$(1^3P_2,(O\bar{O})_{8s}) \to 2g$		0.534	0.446	0.531
$(1^3P_2,(O\bar{O})_{8a})$	$\to (1^3S_1,(O\bar{O})_1)+g$	16.7	12.2	5.14
	$\to (2^3S_1,(O\bar{O})_1)+g$	53.7	27.5	0.0607
$(2^1S_0,(O\bar{O})_{8s}) \to 2g$		37.0	49.2	91.0
$(2^1S_0,(O\bar{O})_{8a})$	$\to (1^3S_1,(O\bar{O})_1)+g$	1.27	0.568	0.479
	$\to (1^1P_1,(O\bar{O})_1)+g$	45.9	24.4	21.2
	$\to (2^3S_1,(O\bar{O})_1)+g$	0.682	0.191	0.0962

TABLE 4a (continued)

		50 GeV	200 GeV	1000 GeV
$(2^3S_1,(O\bar{O})_{8a})$	$\to$ "g" $\to Q\bar{Q}$	0.877	0.957	1.67
	$\to (1^1S_0,(O\bar{O})_1)+g$	0.423	0.189	0.160
	$\to (1^3P_2,(O\bar{O})_1)+g$	25.5	13.6	11.8
	$\to (2^1S_0,(O\bar{O})_1)+g$	0.227	0.0636	0.0321
$(1^1S_0,(O\bar{O})_1)$	$\to \gamma + Z^0$	0.952×10^{-3}	0.0467	0.128
$(2^1S_0,(O\bar{O})_1)$	$\to \gamma + Z^0$	0.00120	0.00768	0.0203
$(1^1S_0,(O\bar{O})_1)$	$\to Z^0\ Z^0$	0	0.458	1.39
$(2^1S_0,(O\bar{O})_1)$	$\to Z^0\ Z^0$	0	0.0759	0.221
$(1^1S_0,(O\bar{O})_1)$	$\to W^+\ W^-$	0	9.64	26.4
$(2^1S_0,(O\bar{O})_1)$	$\to W^+\ W^-$	0	1.59	4.20
$(1^1S_0,(O\bar{O})_{8a})$	$\to \gamma + g$	3.02	5.33	11.8
$(2^1S_0,(O\bar{O})_{8a})$	$\to \gamma + g$	0.734	0.955	1.98
$(1^1S_0,(OO)_{8a})$	$\to Z^0 + g$	0.00172	0.0707	0.163
$(2^1S_0,(O\bar{O})_{8a})$	$\to Z^0 + g$	0.00217	0.0116	0.0259

4b) The $(S\bar{S})$ system

M_S (The mass of the fermion)		50 GeV	200 GeV	1000 GeV
$(1^1S_0,(S\bar{S})_1)$	$\to$ 2g	458	720	1400
$(1^3S_1,(S\bar{S})_1)$	$\to$ 3g	4.87	6.42	10.5
$(1^3P_2,(S\bar{S})_1)$	$\to$ 2g	3.55	3.15	3.62
$(2^1S_0,(S\bar{S})_1)$	$\to$ 2g	90.9	125	229
$(2^3S_1,(S\bar{S})_1)$	$\to$ 3g	0.968	1.11	1.71
$(1^2S_0,(S\bar{S})_8)$	$\to$ 2g	0.00548	0.538×10^{-3}	0.420×10^{-4}
	$\to (1^3S_1,(S\bar{S})_1)+g$	11.6	6.72	5.85
	$\to (1^1P_1,(S\bar{S})_1)+g$	12.0	1.77	0.217
	$\to (2^3S_1,(S\bar{S})_1)+g$	0.0471	0.00332	0.966×10^{-4}
$(1^3S_1,(S\bar{S})_8)$	$\to$ "g" $\to Q\bar{Q}$	5.05	7.64	14.5
	$\to (1^1S_0,(S\bar{S})_1)+g$	3.87	2.24	1.95
	$\to (1^3P_2,(S\bar{S})_1)+g$	6.66	0.982	0.121
	$\to (2^1S_0,(S\bar{S})_1)+g$	0.0157	0.00111	0.322×10^{-4}

TABLE 4b (continued)

		50 GeV	200 GeV	1000 GeV
$(1^1P_1,(S\bar{S})_8)$	$\to (1^1S_0,(S\bar{S})_1)+g$	25.6	21.6	29.2
	$\to (2^1S_0,(S\bar{S})_1)+g$	59.7	32.4	27.6
$(1^3P_1,(S\bar{S})_8)$	$\to$ 2g	0.869	0.771	0.887
	$\to (1^3S_1,(S\bar{S})_1)+g$	25.6	21.6	29.2
	$\to (2^3S_1,(S\bar{S})_1)+g$	59.7	32.4	27.6
$(2^1S_0,(S\bar{S})_8)$	$\to$ 2g	0.00109	0.936×10^{-4}	0.683×10^{-5}
	$\to (1^3S_1,(S\bar{S})_1)+g$	1.79	0.912	0.810
	$\to (1^1P_1,(S\bar{S})_1)+g$	68.8	38.9	35.6
	$\to (2^3S_1,(S\bar{S})_1)+g$	1.16	0.362	0.196
$(2^3S_1,(S\bar{S})_8)$	$\to$ "g" $\to Q\bar{Q}$	1.16	1.37	2.47
	$\to (1^1S_0,(S\bar{S})_1)+g$	0.596	0.304	0.270
	$\to (1^3P_2,(S\bar{S})_1)+g$	38.2	21.6	19.8
	$\to (2^1S_0,(S\bar{S})_1)+g$	0.388	0.121	0.0652
$(1^1S_0,(S\bar{S})_1)$	$\to \gamma + Z^0$	0	0.0486	0.132
$(2^1S_0,(S\bar{S})_1)$	$\to \gamma + Z^0$	0.00110	0.00797	0.0207
$(1^1S_0,(S\bar{S})_1)$	$\to Z^0Z^0$	0	0.476	1.44
$(2^1S_0,(S\bar{S})_1)$	$\to Z^0Z^0$	0	0.0787	0.226
$(1^1S_0,(S\bar{S})_1)$	$\to W^+W^-$	0	10.0	27.4
$(2^1S_0,(S\bar{S})_1)$	$\to W^+W^-$	0	1.65	4.29
$(1^1S_0,(S\bar{S})_8)$	$\to \gamma + g$	4.41	7.82	17.4
$(2^1S_0,(S\bar{S})_8)$	$\to \gamma + g$	0.979	1.37	2.93
$(1^1S_0,(S\bar{S})_8)$	$\to Z^0 + g$	0	0.0982	0.225
$(2^1S_0,(S\bar{S})_8)$	$\to Z^0 + g$	0.00264	0.0161	0.0352

We should note here that the two gluon decay width of color-singlet 1^1S_0 state is about several hundreds MeV. This is larger than that for gluinonium. This difference comes mainly from the Majorana factor 1/2, which should be involved for the gluinonium case. The $^3S_1$ states for both $(O\bar{O})_{\underline{1}}$ and $(S\bar{S})_{\underline{1}}$ are rather narrow. In addition, since the bound states of the onium $(O\bar{O})_{\underline{1}}$ (or $(S\bar{S})_{\underline{1}}$) are much lighter than those of the hybrids $e.g.((O\bar{O})_{\underline{8}}g)_{\underline{1}}$ (or $((S\bar{S})_{\underline{8}}g)_{\underline{1}}$), the hybrid states fall into the corresponding onium states by emitting one gluon will have a presice nonzero possibility, thus we expect a rich monochromatic gluon spectrum. It is also due to such transitions that many of hybrid levels become broad while the lower onium levels remain comparatively narrow.

Having the gluon and the quark decay formula, it is easy to obtain the production cross section formula for hadron collider through gluon fusion or quark annihilation. We will discuss this in detail elsewhere. Here we only point out that the production cross sections are certainly quite big, especially, through the inverse process of $^1S_0$ decay into two gluons, two gluon fusion mechanism.

If the exotic fermions carry electric charge and/or form one extra weak doublet, then the onia and the hybrids may decay into photon(s) and/or intermidiate boson(s) W and Z if its mass falls above the threshold, otherwise decay into virtual W or Z as usual weak β-decay. Since the electroweak properties of the exotic colored fermions are model dependent we just write down the possible decay formulas with some parameters, which need to be determined by a specific model.

i) For the onia, we have:

a). The two photon decay

$$\Gamma\left[0^{-+}_{\underline{1}}\left((O\bar{O})_{\underline{1}} \text{ or } (S\bar{S})_{\underline{1}}\right) \to 2\gamma\right] = 4B\alpha^2Q^4\frac{|R(0)|^2}{M^2} , \tag{45}$$

here 0^{-+} means J^{pc}, Q the charge (in unit e) of the constituents (O or S) and $M = 2m_O$ or $2m_S$, $B = 8$ or 6 for $(O\bar{O})_{\underline{1}}$ or $(S\bar{S})_{\underline{1}}$ respectively, α is the constant of fine structure.

b). The photon and Z^0 decay

$$\Gamma\left[0^{-+}_{\underline{1}}\left((O\bar{O})_{\underline{1}} \text{ or } (S\bar{S})_{\underline{1}}\right) \to \gamma + Z^0\right] = \frac{8B\alpha^2Q^2}{sin^2\theta_w cos^2\theta_w}$$

$$\left(\frac{1}{2}T_3 - Qsin^2\theta_w\right) \cdot \frac{|R(0)|^2}{M^2}\left(1 - \frac{m_z^2}{M^2}\right)^{\frac{1}{2}}, \tag{46}$$

here T_3—the weak isospin is z axis. θ_w—the weinberger angle.

c). The two Z^0 decay

$$\Gamma\left[0^{-+}_{\underline{1}}\left((O\bar{O})_{\underline{1}} \text{ or } (S\bar{S})_{\underline{1}}\right) \to Z^0Z^0\right] = \frac{4B\alpha^2}{sin^4\theta_w cos^4\theta_w}$$

$$\cdot\left(\frac{1}{2}T_3^2 - T_3Qsin^2\theta_w + Q^2sin^4\theta_w\right)^2\frac{|R(0)|^2}{M^2}$$

$$\cdot\left(1 - \frac{4m_z^2}{M^2}\right)^{\frac{1}{2}}\frac{(M^2 - m_z^2)(M^2 - 3m_z^2)}{(M^2 - 2m_z^2)^2}, \tag{47}$$

d). The W^-W^+ pair decay

$$\Gamma\left[0^{-+}_{\underline{1}}\left((O\bar{O})_{\underline{1}} \text{ or } (S\bar{S})_{\underline{1}}\right) \to W^+W^-\right] = \frac{\alpha^2B}{2sin^4\theta_w}\frac{|R(0)|^2}{M^2}$$

$$\frac{(M^2 - 3m_w^2)(M^2 - m_w^2)}{(M^2 - 3m_w^2)^2}\left(1 - \frac{4m_z^2}{M^2}\right)^{\frac{1}{2}}. \tag{48}$$

ii) For the hybrids, we have:

a). The photon and gluon decay

$$\Gamma\left[0^{-+}_{\underline{8}}\left((O\bar{O})_{\underline{8}} \text{ or } (S\bar{S})_{\underline{8}}\right) \to \gamma + g\right] = 8Q^2\alpha\alpha_s\frac{|R(0)|^2}{M^2}B_c , \tag{49}$$

here $B_c = 3$ for $(O\bar{O})_{\underline{8}}$, $B_c = \frac{5}{3}$ for $(S\bar{S})_{\underline{8}}$.

b). The Z^0 and gluon decay

$$\Gamma\left[0^{-+}_{\underline{8}}\left((O\bar{O})_{\underline{8}} \text{ or } (S\bar{S})_{\underline{8}}\right) \to Z^0 + g\right] = 8\frac{\alpha\alpha_sQ^2}{sin^2\theta_w cos^2\theta_w}$$

$$\left(\frac{1}{2}T_3 - Qsin^2\theta_w\right)^2\frac{|R(0)|^2}{M^2}\left(1 - \frac{m_z^2}{M^2}\right)^{\frac{1}{2}} \tag{50}$$

Here we restrict ourselves to consider $J^{PC} = 0^{-+}$ only as some examples, so as to have some idea on these decays.

IV. Summary

According to our discussions and the numerical results, we would emphasize several points here.

First of all it is easy to see that an exotic colored fermion and antifermion system has plentiful bound states, in addition to its onium. It has several series of hybrids because the exotic fermion is a high demension representation of color. For example, for $(O\bar{O})$ system we have an onium, $((O\bar{O})_{\mathbf{8}_s},g)_{\mathbf{1}}$, $((O\bar{O})_{\mathbf{8}_a},g)_{\mathbf{1}}$, $((O\bar{O})_{\mathbf{10}},gg)_{\mathbf{1}}$, $((O\bar{O})_{\overline{\mathbf{10}}},gg)_{\mathbf{1}}$ and $((O\bar{O})_{\mathbf{27}},gg)_{\mathbf{1}}$. In this paper we discussed $((O\bar{O})_{\mathbf{10}},gg)_{\mathbf{1}}$, $((O\bar{O})_{\overline{\mathbf{10}}},gg)_{\mathbf{1}}$ and $((O\bar{O})_{\mathbf{27}},gg)_{\mathbf{1}}$ less than $((O\bar{O})_{\mathbf{8}_a},g)_{\mathbf{1}}$ and $((O\bar{O})_{\mathbf{8}_s},g)_{\mathbf{1}}$. However, for the usual quark -antiquark system, we have only two, one onium and one hybrid $((q\bar{q})_{\mathbf{8}},g)$. Here not only in the onium but also in some hybrids *e.g.* $(O\bar{O})_{\mathbf{8}_a},(O\bar{O})_{\mathbf{8}_s}$, the interactions between the two constituents are attractive even at short distances (see Fig 1.), so that there are some states whose binding energies are negative (relatively stable). Thus we have more reasons to expect clear experimental evidence for the spectra of $(O\bar{O})_{\mathbf{8}_a}$ and $(O\bar{O})_{\mathbf{8}_s}$ as well as those for its onium will be observed if the exotic colored fermions do exist.

Owing to the heavy masses of the exotic colored fermions, the exotic fermions in hybrids are located at the center of the bag essentially and the hybrid bag is closed to a sphere for low lying states, especially for those kinds of hybrids as $(O\bar{O})_{\mathbf{8}_a},(O\bar{O})_{\mathbf{8}_s}$ and $(S\bar{S})_{\mathbf{8}}$ with attractive interaction in whole range. This is consistent with the starting point that, for simplicity, we ignored the bag deformation when we computed the hybrid effective potential. Although we also computed some states with the molecular-like model in section 2, the spherical model should be considered to be more realizable, and the calculations on those kinds of hybrids as $(O\bar{O})_{\mathbf{8}_a},(O\bar{O})_{\mathbf{8}_s}$ and $(S\bar{S})_{\mathbf{8}}$ for low lying state are more convincing.

The $(O\bar{O})$ system has some interesting aspects. Being in an octet for the constituent fermion O, the gauged coupling of it to gluon is antisymmetric (adjoin representation) so that $(O\bar{O})_{\mathbf{1}}\ J^{PC}=1^{--}$ state can not decay into 3 gluons under first order approximation.[4]. Hence the $(O\bar{O})_{\mathbf{1}}\ \ J^{PC}=1^{--}$ will have longer lifetime than that of $(S\bar{S})_{\mathbf{1}}\ \ J^{PC}=1^{--}$. Moreover, there are two series $(O\bar{O})_{\mathbf{8}_a}$ and $(O\bar{O})_{\mathbf{8}_s}$ hybrids and under the lowest order approximation they are degenerated but their decay behavior is quite different as pointed in Sec.3. So in principle this point might be used to identify the color multiplet as an octet, but we should note that if higher order corrections are considered the degeneration of $(O\bar{O})_{\mathbf{8}_a}$ and $(O\bar{O})_{\mathbf{8}_s}$ will be removed, for example, by considering self energy splitting due to gluon(s) annihilation, owing to that $(O\bar{O})_{\mathbf{8}_s}$ starts with two gluon annihilation but $(O\bar{O})_{\mathbf{8}_a}$ with one, then the degeneration must be melted. If higher order corrections are considered, not only the degeneration will be melted but also some new phenomena will occur, for example, the energy level splitting upto fine and hyperfine structures as well as possible mixing among the states etc,. All of these we have not touched in this paper, although these may make some changes on the spectra and various transitions, we will discuss these elsewhere.

[4]Some authors have pointed this out [33],[34]

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Figure Captions

Fig. 1: The effective potentials between the constituents fermion and antifermion in onia and hybrids.

Fig. 2: The lowest order diagram of the two gluon decay channel for onia and hybrids.

Fig. 3: The lowest order diagram of the quark pair decay channel for onia and hybrids.

Fig. 4: The lowest order diagram of the three gluon decay channel for onia and hybrids.

Fig. 5: The lowest order diagram of the emitting one gluon channel between a hybrid and a onium.

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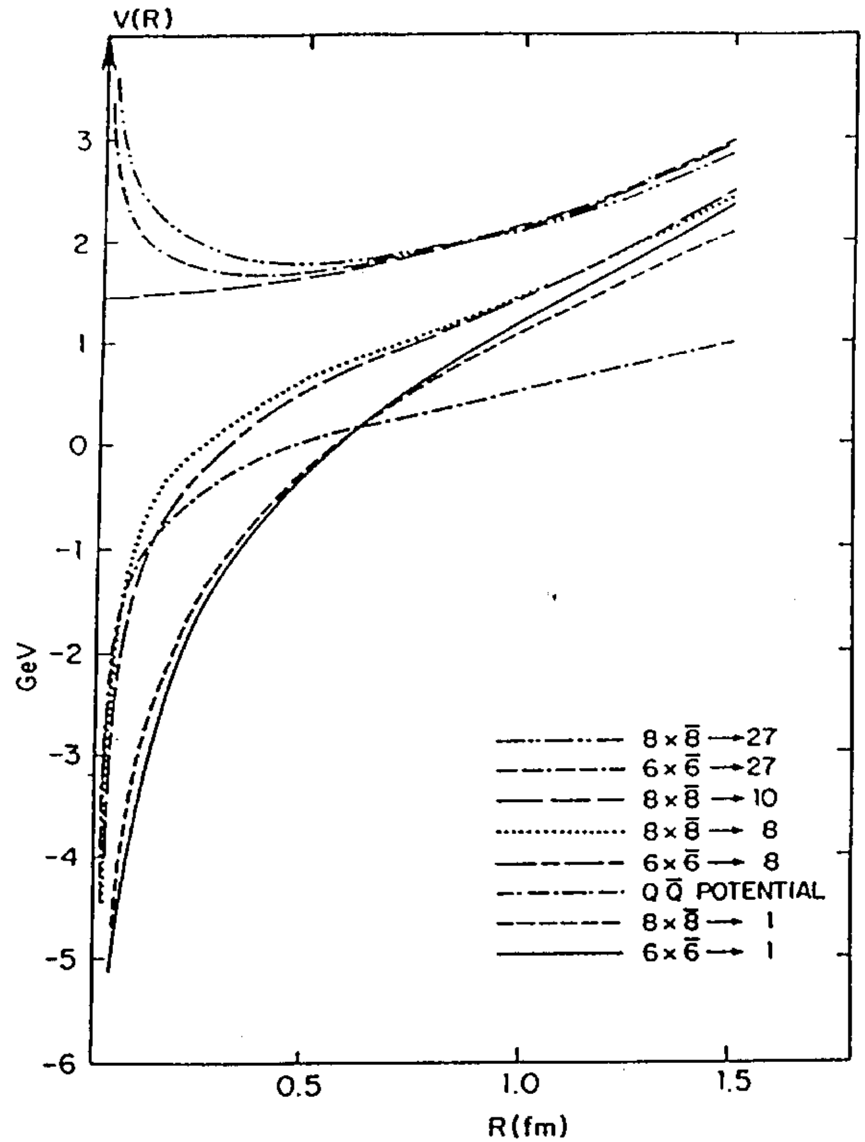

Fig.1

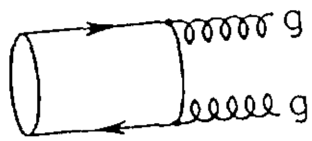

Fig.2

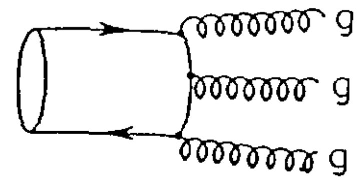

Fig.4

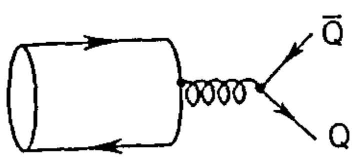

Fig.3

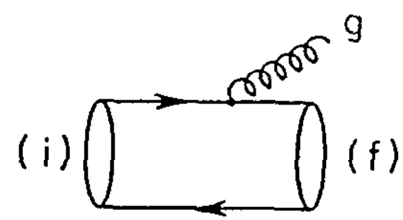

Fig.5